# ENDOGENOUS NOISE TRADERS

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SSE/EFI Working Paper Series in Economics and Finance
No 644

December 5, 2006

Abstract. We construct a parsimonious model of a financial market where the marginal investor is an endogenous noise trader. Such a trader anticipates that future shocks may force him to exit his position. In compensation he requires a higher return. We show that the original seller of the asset pays the required return. This can only be optimal if the seller has access to an investment opportunity that gives a sufficiently high return, compared to the noise trader's investment opportunities. We also show that, if the noise trader expects to get informative signals, the required return does not necessarily decrease, as claimed in the earlier literature.

Keywords: Market microstructure, no-trade theorems, adverse selection

JEL Classification: G14.

## 1. Introduction

Bagehot (1971) was one of the first to note that market makers face a problem when trading with informed traders. Since informed traders can choose not to trade if the prices do not suit them, market makers will never gain from trading with them - and might sometimes lose. This adverse selection problem may lead to market making not being viable, and markets may break down. However, Bagehot[1] also suggested that exogenously motivated traders, or so called noise traders, could provide the market maker with enough gains to compensate for the losses on informed traders.

Noise traders have ever since played an important role in the market microstructure literature. Indeed Black (1986) concluded that "[n]oise makes trading in financial markets possible". In the model of Copeland and Galai (1983), and in the dynamic extension of Glosten and Milgrom (1985), they are needed for the market maker to finance losses to informed traders. In Kyle (1985) noise traders provide camouflage for a monopolistic informed trader. Noise traders, under the guise of an exogenously changing supply, ensure that prices are somewhat inefficient in Grossman and Stiglitz (1980). This allows for informed traders to recover information costs.[2]

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*I am grateful to the Wallenberg Foundation for financial support. I am also grateful for comments from Cédric Argenton, Milo Bianchi, Andrei Simonov, and Jörgen Weibull. Any errors are my own.

[1]The article was written pseudonymously by Jack Treynor.

[2]Shleifer and Summers (1990) give an outline of the noise trader approach to finance. Recent surveys of the market microstructure literature are O'Hara (1995), Madhavan (2000), Brunnermeier (2001), Stoll (2003), and Biais, Glosten, and Spatt (2005).

The need for noise traders in models of financial markets can also be understood as a way of side-stepping the various no-trade theorems, among which Milgrom and Stokey (1982) and Tirole (1982) are the most well-known. The principle behind these theorems follows from Aumann (1976): "If two people have the same priors, and their posteriors for an event A are common knowledge, then these posteriors are equal." Translated into the world of financial markets, this means that, given the above assumptions, these two people must agree on the price of an asset. As a result, they may trade - even if they start off from a Pareto optimal allocation - but they will be indifferent between trading and not trading. Thus, as soon as any costs of trading are introduced, there will be no trade. At least one participant will lose from trading, and will then prefer not to trade. The costs may be transaction costs, information costs, or, as in Milgrom and Stokey (1982), remuneration for risk.

A problem with the noise trader approach is that noise traders systematically lose money. As discussed in Dow and Gorton (2006), this has led to a literature trying to endogenize noise traders as rational agents. Various approaches have been considered. Diamond and Verrecchia (1981) suggest that noise traders may trade for insurance reasons. De Long, Shleifer, Summers, and Waldman (1990) consider the possibility that arbitrageurs have a limited time horizon. This results in a limited arbitrage that makes it possible for noise traders to survive. Shleifer and Vishny (1997) elaborate on this idea and consider agency problems. They consider a situation where an arbitrageur borrows money from an uninformed investor. As the investor is uninformed, he tries to conjecture whether the arbitrageur's positions are sound or not by observing returns. A temporary shock can then lead to the investor recalling the money, although the arbitrageur's position is fundamentally sound. Another approach, also based on the agency problem, has been developed by Dow and Gorton (1997). They note that a fund manager might trade excessively to appear informed to his investors. This excessive trade results in a systematic loss that would correspond to the loss of noise traders.

In this paper, we will consider yet another approach, based on the notion of *required return*. According to this approach, the noise trader anticipates his future trading losses when selling the asset, and thus demands a discount when buying the asset. The argument can be traced back to Amihud and Mendelson (1986) who argued that, since the bid-ask spread is a trading cost, it should be positively correlated with the expected return. Gârleanu and Pedersen (2004) further developed the argument by arguing that if the bid-ask spread was caused by adverse selection, as claimed by Copeland and Galai (1983), then traders would demand a higher return if they expected to exit their position due to a liquidity shock, but a lower return if they expected to exit due to private information. As a result, Gârleanu and Pedersen argued, the two effects can cancel each other, and only when the liquidity signal and the informational signal are in conflict will an actual cost be incurred.

We extend the model of Copeland and Galai (1983) with the aim to construct the simplest model possible to endogenize noise traders. In section 2, we illustrate the adverse selection problem in a simple model with risk neutral players, but where informative signals are costly. We show that without noise traders, no costly information will be acquired. However, if exogenous noise traders are introduced, then information costs can be covered. In section 3, we extend the model to endogenize the noise traders.

Anticipating that they may have to exit their position at a loss, they will only enter it at a discount. Furthermore, we can analyze the effect of the competitive structure among informed traders. It turns out that if the informed trader is a monopolist, then the seller will have to finance his profit. However, a monopolistic informed trader will choose a lower precision than if he were competitive. As a result, the seller does not have to compensate the noise trader to the same extent. Thus, the unambiguous result, if information costs are convex in the precision, is that the seller's costs increase if the informed trader is competitive. In section 4, we allow the noise trader to be partly informed, just as in Gârleanu and Pedersen (2004). We consider two cases. In the first case, the noise trader becomes informed by buying the asset. Then Gârleanu and Pedersen's conjecture is always true - i.e. the required return falls. In the second case, the noise trader is informed already before buying the asset. Despite the fact that he has not actually received the informative signal - he only knows that he will receive it - this influences his opportunity cost. As a result he is not ready to pay as high a price as he would as uninformed. Thus, Gârleanu and Pedersen's conjecture will only be true for some parameter values.

## 2. Adverse selection

**2.1. The model.** The adverse selection problem on financial markets can be illustrated as follows. Let us assume that we have an informed trader $(IT)$ and a competitive market maker $(M)$,[3] and that the timing of the game is as follows:

1. Nature draws a fundamental value for the asset $v \in \{0, 1\}$ with equal probability. The realization is not observed by anybody.
2. The informed trader chooses a precision $\lambda \in [1/2, 1]$ for a signal to be received in 4. The signal costs $c(\lambda)$, where $c\left(\frac{1}{2}\right) = c'\left(\frac{1}{2}\right) = 0$, and $c'(\lambda), c''(\lambda) > 0$.
3. The competitive market maker observes $\lambda$ and announces ask and bid prices $p_A, p_B \in [0, 1]$.
4. The informed trader gets the signal $s \in \{0, 1\}$, where
$$\begin{aligned} \Pr(v = 1 \mid s = 1) &= \Pr(v = 0 \mid s = 0) = \lambda \\ \Pr(v = 0 \mid s = 1) &= \Pr(v = 1 \mid s = 0) = 1 - \lambda. \end{aligned}$$
He observes the bid-ask prices and chooses whether to buy, sell, or do nothing. His strategy set is thus $S_{IT} = \{B, S, N\}$.
5. The asset expires and all trades clear at the fundamental value $v$. More precisely, the market maker and the informed trader reverse their trade, if there was a trade, at the price $v$.

The players are rational and risk neutral. There is no time discounting and the reservation value is 0. We allow for both long and short positions, and for futures contracts, if a prospective short seller can not borrow an asset.

[3] We do not model why the market maker is competitive. However, as discussed in Glosten and Milgrom (1985) it may be a result of competition with other limit orders or with another market maker at the same, or another exchange.

**2.2. Analysis.** The only subgame perfect Nash equilibrium in this game is for the informed trader to not pay an information cost and thus receive an uninformative signal. To see this, let us solve the game by backward induction.

In 4, IT's conditional payoffs are

$$
\begin{aligned}
E\left[\pi_{IT} \mid s=1, B\right] &= \lambda\left(1-p_A\right)+(1-\lambda)\left(0-p_A\right)-c \\
E\left[\pi_{IT} \mid s=1, S\right] &= \lambda\left(p_B-1\right)+(1-\lambda)\left(p_B-0\right)-c \\
E\left[\pi_{IT} \mid s=1, N\right] &= -c \\
E\left[\pi_{IT} \mid s=0, B\right] &= \lambda\left(0-p_A\right)+(1-\lambda)\left(1-p_A\right)-c \\
E\left[\pi_{IT} \mid s=0, S\right] &= \lambda\left(p_B-0\right)+(1-\lambda)\left(p_B-1\right)-c \\
E\left[\pi_{IT} \mid s=0, N\right] &= -c.
\end{aligned}
$$

Thus, if $IT$ gets the signal $s=1$, then with probability $\lambda$ it is correct, and by buying he would get the payoff $\left(1-p_A\right)$. With probability $1-\lambda$ it is incorrect and then he would get the payoff $\left(0-p_A\right)$. The information costs are sunk, so if $IT$ does not trade, then his payoff is $-c$.

Thus, $IT$ weakly gains from buying if

$$p_A \leq \lambda \quad \text{and} \quad s=1, \tag{1}$$

and from selling if

$$p_B \geq 1-\lambda \quad \text{and} \quad s=0. \tag{2}$$

In 3, the competitive market maker will set bid-ask prices so that they by themselves result in an expected profit of zero. He takes the reversed position of the informed trader - half of the time on each side. Thus, the bid-ask prices must satisfy

$$-\frac{1}{2}\left(\lambda-p_A\right) = 0 \tag{3}$$

$$-\frac{1}{2}\left(p_B-(1-\lambda)\right) = 0. \tag{4}$$

Reshuffling, we get the equilibrium bid-ask prices

$$p_A^* = \lambda \tag{5}$$

$$p_B^* = 1-\lambda. \tag{6}$$

It follows immediately that these bid-ask prices satisfy conditions $(1)-(2)$. In other words, if the market maker sets prices so as to achieve zero profits when trading, then the informed trader will trade according to his signal.[4]

In 2, IT's expected profit is

$$E\left[\pi_{IT}\right]=\lambda-\frac{1}{2}-\frac{1}{2}\left(p_A-p_B\right)-c(\lambda). \tag{7}$$

[4] Anticipating $IT's$ trading decision in 4, the market maker can also achieve zero profits by setting $p_A \geq \lambda$ and $p_B \leq 1-\lambda$, where at least one inequality is strict. Then profits will be zero for the simple reason that $IT$ will not trade on at least one side. Since we interpret the market maker's zero profit condition as resulting from competition, we disregard this possibility.

Inserting the bid-ask prices, we get

$$E\left[\pi_{IT}\right] = -c\left(\lambda\right). \tag{8}$$

Thus, the only information cost that $IT$ can cover is $c = 0$. The reason is that $M$ will never announce prices so that he makes a loss on expectation, but then $IT$ can never cover strictly positive information costs.

**Exogenous noise traders.** The standard approach to solving this dilemma is to assume that some traders trade for exogenous reasons. This type of traders are usually called liquidity traders or noise traders. In this paper we have chosen the term noise traders.

Let us assume that noise traders trade with the probability $\alpha > 0$ on the ask side, and with probability $\beta > 0$ on the bid side. Then the competitive market maker faces a different problem in 3. The bid-ask prices must now satisfy

$$\alpha\left(p_A - \frac{1}{2}\right) - \frac{1}{2}\left(\lambda - p_A\right) = 0 \tag{9}$$

$$\beta\left(\frac{1}{2} - p_B\right) - \frac{1}{2}\left(p_B - \left(1-\lambda\right)\right) = 0. \tag{10}$$

The equilibrium bid-ask prices are

$$p_A = \frac{\alpha + \lambda}{2\alpha + 1} \tag{11}$$

$$p_B = \frac{\beta + 1 - \lambda}{2\beta + 1}. \tag{12}$$

Again it is easy to see that these bid-ask prices satisfy conditions $(1) - (2)$. Furthermore, in 2, IT's expected profit is

$$E\left[\pi_{IT}\right] = \lambda - \frac{1}{2} - \frac{1}{2}\left(\frac{\alpha + \lambda}{2\alpha + 1} - \frac{\beta + 1 - \lambda}{2\beta + 1}\right) - c\left(\lambda\right). \tag{13}$$

It is weakly positive if

$$\frac{\left(2\lambda - 1\right)}{2}\frac{\left(\alpha + \beta + 4\alpha\beta\right)}{\left(2\beta + 1\right)\left(2\alpha + 1\right)} \geq c\left(\lambda\right). \tag{14}$$

Thus, if only $\lambda > 1/2$, i.e. if the signal is informative, then a positive information cost can be covered - provided it is sufficiently low.

Thus the introduction of exogenous noise traders makes it possible to finance costly information acquisition. On the other hand, since these noise traders trade for exogenous reasons, we have pushed the actual source of financing outside the model. The objective with the concept of required return is to bring it inside the model. In the next section we will show how that may be done.

## 3. Required return

The main idea with the concept of required return is that if the noise trader anticipates that he might have to exit the asset at a cost, then he will only enter at a discount. Note, however, that this implies that only the noise trader on the bid side in the previous example can be endogenized. A noise trader on the ask side, if he is rational, would still sell for some exogenous reason.

**3.1. The model.** Let us imagine a game with four representative types; a seller $(S)$, a noise trader $(NT)$, an informed trader $(IT)$, and a competitive market maker $(M)$. The timing of the model is as follows.

1. The seller gives the noise trader a take-it-or-leave-it offer to buy one unit of an asset for $p_0 \in [0,1]$. The seller will use the funds to finance a project with an expected return $E[r_S] > 0$.
2. The noise trader decides whether to accept the seller's offer or not.
3. Nature draws a fundamental value for the asset $v \in \{0,1\}$ with equal probability. The realization is not observed by anybody.
4. The informed trader chooses a precision $\lambda \in [1/2, 1]$ for the signal that he will receive in 6. The signal costs $c(\lambda)$, where $c\left(\frac{1}{2}\right) = c'\left(\frac{1}{2}\right) = 0$, and $c'(\lambda), c''(\lambda) > 0$.
5. The competitive market maker observes $\lambda$ and announces ask and bid prices $p_A, p_B \in [0,1]$.
6. The informed trader gets the signal $s \in \{0,1\}$, where
$$\begin{aligned}
\Pr(v=1 \mid s=1) &= \Pr(v=0 \mid s=0) = \lambda \\
\Pr(v=0 \mid s=1) &= \Pr(v=1 \mid s=0) = 1-\lambda.
\end{aligned}$$
He observes the bid-ask prices and chooses whether to buy, sell, or do nothing. His strategy set is thus $S_{IT} = \{B, S, N\}$. With probability $\beta$ the noise trader gets a shock and must sell for $p_B$. With probability $1-\beta$ the noise trader gets no shock and keeps the asset.
7. The asset expires and all trades clear at the fundamental value $v$. More precisely, the market maker and the informed trader reverse their trade, if there was a trade, at the price $v$. The asset is bought by the seller for the price $v$.

Thus, compared to the previous model we have added two players, who each make decisions before trading occurred in the previous model.

We have an endogenous noise trader. He might receive a shock in 6, which hinders him to keep the asset to maturity. However, he anticipates this shock when deciding at which price he is ready to buy the asset in 2.

We also have a seller of the asset, who might sell the asset to the noise trader in 2. The seller can be interpreted either as the issuer of the asset, for example at an IPO, or as any owner of the asset who considers selling the asset to use the funds for some other investment opportunity.[5]

As before, all players are rational and risk neutral. There is no time discounting and the reservation value is 0 - except for $S$. We allow for both long and short positions, and for futures contracts, if a prospective short seller can not borrow an asset.

**3.2. Analysis.** Solving the model by backward induction, we now note that the market maker will announce the bid-ask prices

$$p_A^* = \lambda \tag{15}$$

$$p_B^* = \frac{\beta + 1 - \lambda}{2\beta + 1}. \tag{16}$$

It is easy to verify that these bid-ask prices satisfy conditions $(1) - (2)$.

If $IT$ is a monopolist, then he maximizes profits with respect to $\lambda$. Inserting equations (15) and (16) and taking the first order condition, we can conclude that the equilibrium precision is given by

$$\lambda^* = \lambda_{MON} = c'^{-1}\left(\frac{\beta}{2\beta + 1}\right).$$

On the other hand. if the profits are pushed to zero, for example due to competition, then the equilibrium precision is given by $\lambda^* = \lambda_{COMP}$, where $\lambda_{COMP}$ satisfies

$$\frac{1}{2}\left(2\lambda_{COMP} - 1\right)\frac{3\beta + 2}{2\beta + 1} = c\left(\lambda_{COMP}\right). \tag{17}$$

Since $c(\lambda)$ is convex, we know that $\lambda_{COMP} \geq \lambda_{MON}$.

In 2, the expected profit of $NT$ is

$$E\left[\pi_{NT}\right] = \beta p_B^* + (1 - \beta) E[v] - p_0. \tag{18}$$

He will thus accept any offer that satisfies

$$p_0 \leq \beta p_B^* + (1 - \beta) E[v], \tag{19}$$

and reject other offers.

Moving back to 1, the seller's expected profit is

$$E\left[\pi_S\right] = \left(1 + E\left[r_S\right]\right) p_0 - E[v].$$

Thus, he will only give offers that satisfy

$$p_0 \geq \frac{E[v]}{1 + E\left[r_S\right]}.$$

[5] The fact that the seller possibly buys back the asset in 7 is simply a technical device to calculate the expected profit from his initial sale.

Thus for

$$\frac{E[v]}{1+E[r_S]} \leq \beta p_B^* + (1-\beta) E[v],$$

the seller will give the noise trader an offer that he will accept. Reshuffling, and inserting (16), and $E[v] = 1/2$, the condition becomes

$$\frac{(2\lambda^* - 1)\beta}{1 + 2\beta - \beta(2\lambda^* - 1)} \leq E[r_S]. \tag{20}$$

If condition (20) is satisfied, then the seller uses his first mover advantage and gives the offer $p_0^*$, where

$$p_0^* = \beta p_B^* + (1-\beta) E[v]. \tag{21}$$

The equilibrium required return, or equivalently, the seller's cost of capital, is thus

$$\begin{aligned} r^* &= \frac{E[v] - p_0^*}{p_0^*} \\ &= \frac{(2\lambda^* - 1)\beta}{3\beta - 2\beta\lambda^* + 1}. \end{aligned}$$

If condition (20) is not satisfied, then the seller can not give the noise trader an offer he will accept. In this case, the asset will thus not be sold. Since the left hand side is strictly positive, it is not enough if $E[r_S]$ simply is strictly positive - it must be sufficiently high. Thus, the seller and the noise trader must have sufficiently asymmetric investment opportunities.

Calculating the sensitivity of the required return to the precision of the signal, we get

$$\frac{\delta r^*}{\delta \lambda^*} = \frac{2(2\beta+1)\beta}{(2\beta\lambda^* - 3\beta - 1)^2} > 0. \tag{22}$$

In other words, a more competitive informed trader implies that the required return increases. Thus, although the seller may have to pay the monopolist informed trader's positive profits, he still prefers that to having to pay for the increased adverse selection cost when the informed traders is competitive.

Also note that the bid-ask spread depends on the precision $\lambda$, but only indirectly on the information cost. This complements the result of Gârleanu and Pedersen (2004) that transaction costs increase the bid-ask spread.

### 3.3. An example of $c(\lambda)$.

Let us explore an example where the cost function is

$$c(\lambda) = \ln\left(\frac{1}{2(1-\lambda)}\right) + 1 - 2\lambda. \tag{23}$$

The cost function is plotted in Figure 1 below. We have also plotted $IT$'s revenues when $\beta = 0$ and when $\beta = 1$. The line $2\lambda - 1$ corresponds to $\beta = 1$. Decreasing $\beta$ leads to a decrease of the slope of the revenue line.

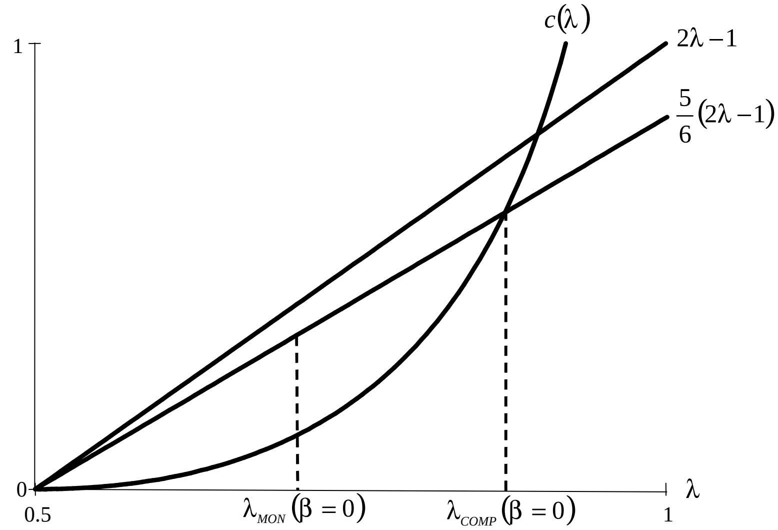


Figure 1: The cost function and $IT$'s revenue when $\beta = 0$ and $\beta = 1$.

Using (23), $\lambda_{MON}$ is given by

$$\lambda_{MON} = c'^{-1}\left(\frac{\beta}{2\beta+1}\right) = \frac{3\beta+1}{5\beta+2}. \tag{24}$$

Note that since

$$\frac{\delta\lambda_{MON}}{\delta\beta} = \frac{1}{(5\beta+2)^2} > 0 \tag{25}$$

$$\frac{\delta\lambda_{MON}}{\delta\beta} = -\frac{10}{(5\beta+2)^3} < 0, \tag{26}$$

$\lambda_{MON}$ is increasing and concave in $\beta$. Figure 2 below shows how the optimal precision changes as a function of $\beta$.

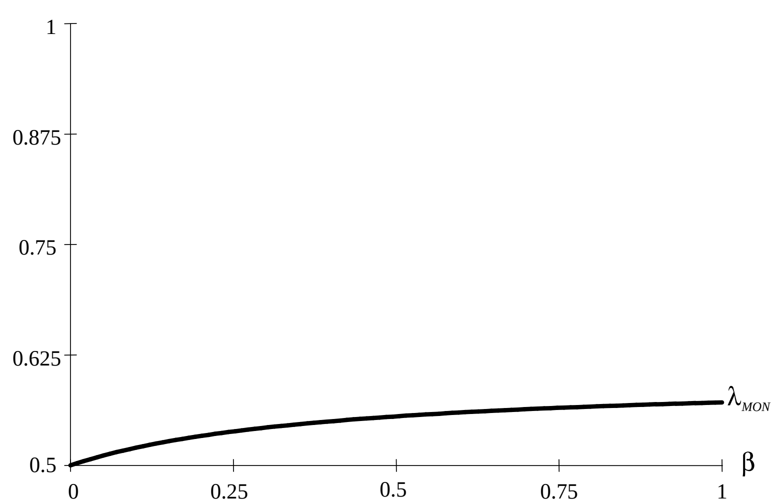


Figure 2: The optimal precision as a function of $\beta$.

Using Equations (15) – (16) we thus get

$$p_B^* = \frac{4\beta + 5\beta^2 + 1}{(5\beta + 2)(2\beta + 1)} \tag{27}$$

$$p_A^* = \frac{3\beta + 1}{5\beta + 2}. \tag{28}$$

It follows that

$$p_0^* = \frac{1}{2}\frac{(3\beta + 1)(3\beta + 2)}{(5\beta + 2)(2\beta + 1)}. \tag{29}$$

Figure 3 shows equilibrium bid-ask prices as $\beta$ increases, as well as $p_0^*$ (the thin downward sloping curve).

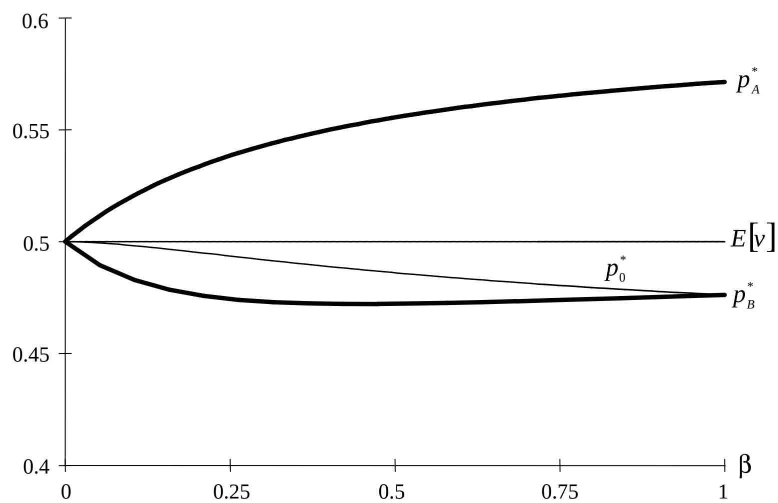


Figure 3: The bid ask spread, $p_0^*$, and $E[v]$ as a function of $\beta$.

The horizontal line $E[v]$ shows the expected value of the asset. Note that the spread is not symmetric around this value. This is because noise traders trade on the bid side, and thus push the bid price upwards.

Thus, as $\beta$ increases, $M$ gets higher trading gains vis-à-vis $NT$. As a result, the possibility to make trading losses vis-à-vis $IT$ increases. $IT$ can thus finance more costly information. As the precision of signals rise, the bid-ask spread also rises. The bid-ask spread as a function of $\beta$ is concave for two reasons. First, since the cost function is convex, an increase in $\beta$ allows a less than proportional increase in $\lambda$ from a cost perspective. Second, as the bid-ask spread increases in $\lambda$, the gain from a higher precision is also less than proportional.

Finally, the required return is

$$r^* = \frac{\beta^2}{(3\beta + 1)(3\beta + 2)}.$$

In figure 4 we plot the required return as a function of $\beta$.

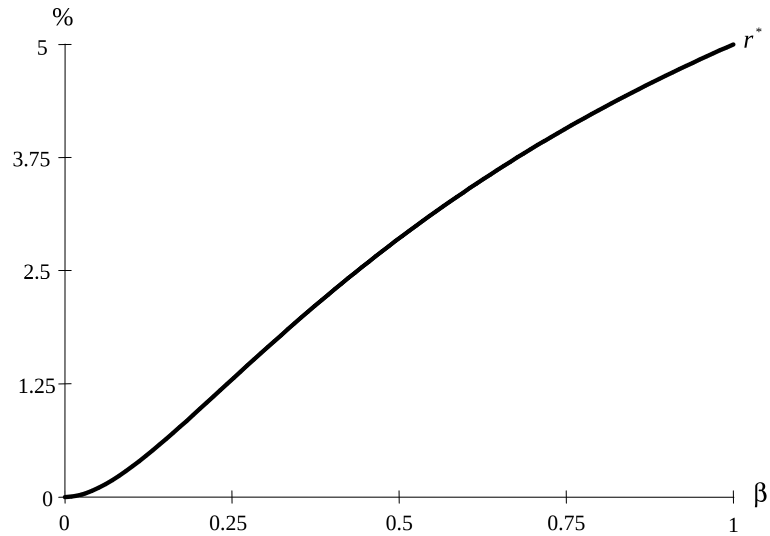


Figure 4: The required return as a function of $\beta$.

## 4. Information structure

Gârleanu and Pedersen (2004) argued that a noise trader that also may receive an information shock will ask for a lower required return. Here we show that this may not always be the case. The reason is that the opportunity cost differs depending on how the information structure is defined.

First, if we assume that the noise trader becomes informed when he buys the asset, then the asset is worth more to him and the required return decreases. This is thus in line with Gârleanu and Pedersen's case. In fact, there are two effects. The first is that the trader benefits from being informed. The other is that if he buys, then he buys at $p_0$, and not at $p_A$. Thus, he avoids the adverse selection pricing when buying.

Second, if we assume that the noise trader knows that he might receive an informational signal even if he does not buy the asset, then the opportunity cost is different. It is no longer zero, but strictly positive. As a result, the price he is ready to pay decreases, and the required return increases. If this outweighs the fact that the trader can avoid the adverse selection pricing when receiving positive signals, then Gârleanu and Pedersen's claim is not valid.

Let us thus define a new player, a partially informed noise trader $(NT, IT)$, who may receive an informational signal with probability $\gamma \in [0,1]$. For the purposes of this argument, we disregard the information cost. We also treat the bid-ask prices $(15)-(16)$ as given.[6]

[6] One interpretation is that the new player $NT, IT$ is atomistic compared to the other representative players. This assumption simplifies the argument, but does not affect it significantly.

**4.1.** ***NT*** **becomes informed if he buys.** In 2, the expected profit of $NT, IT$ is

$$\begin{aligned} E\left[\pi_{NT,IT}\right] &= \beta\left(1-\gamma\right)\left(p_B^*-p_0\right)+ \\ & \left(1-\beta\right)\left(1-\gamma\right)\left(\frac{1}{2}-p_0\right)+ \\ & \beta\gamma\left(\frac{1}{2}\left[\lambda-p_0\right]+\frac{1}{2}\left[\lambda\left(p_B^*-0\right)+\left(1-\lambda\right)\left(p_B^*-1\right)+\left(p_B^*-p_0\right)\right]\right)+ \\ & \left(1-\beta\right)\gamma\left(\frac{1}{2}\left[\lambda-p_0\right]+\frac{1}{2}\left[\lambda\left(p_B^*-0\right)+\left(1-\lambda\right)\left(p_B^*-1\right)+\left(p_B^*-p_0\right)\right]\right) \end{aligned}$$

He will thus accept any offer that satisfies

$$p_0 \leq \bar{p}_0 = \left(1-\gamma\right)\left(\beta p_B^* + \left(1-\beta\right)\frac{1}{2}\right) + \gamma\left(\lambda - \frac{1}{2} + p_B^*\right) \tag{30}$$

The required return is thus lower in this case if $\bar{p}_0 \geq p_0^*$. Inserting (16) , (21) , and (30) , we get

$$\frac{3}{2}\frac{\left(2\lambda-1\right)\beta\gamma}{\left(2\beta+1\right)} \geq 0, \tag{31}$$

which is always the case.

**4.2.** ***NT*** **is informed even if he does not buy.** In this case, the opportunity cost changes as well. For $NT, IT$ to buy the asset, we must have

$$E\left[\pi_{NT,IT}\right] \geq E\left[\pi_{IT}\right].$$

He will thus accept any offer that satisfies

$$p_0 \leq \hat{p}_0 = \left(1-\gamma\right)\left(\beta p_B^* + \left(1-\beta\right)\frac{1}{2}\right) + \gamma\left(\lambda - \frac{1}{2} + p_B^*\right) - \left(\lambda - \frac{1}{2} - \frac{1}{2}\left(p_A^* - p_B^*\right)\right). \tag{32}$$

The required return is thus lower in this case if $p_0^* < \hat{p}_0$. Inserting (15) – (16) , (21) , and (32) , we get

$$0 < \frac{1}{2}\frac{\left(3\gamma-1\right)\left(2\lambda-1\right)\beta}{\left(2\beta+1\right)},$$

which only is true if

$$\frac{1}{3} < \gamma.$$

Thus, only if the probability of being informed is sufficiently high will the required return be lower if the trader is informed before buying the asset. Note that the benefit stems not from being informed *per se*, but rather from the fact that if the trader gets a positive signal, then he has already bought in 2, which makes it possible for him to avoid the adverse selection pricing in 6.

## 5. Conclusion

We have aimed to create a simple way to extend Copeland and Galai (1983) to incorporate endogenous noise traders. We have shown that the concept of required return implies that the adverse selection cost ultimately is paid by the seller of the asset. It is thus part of his cost of capital. We have framed the model so that the seller is any owner of the asset. However, he could also be interpreted as the original issuer of the asset. We showed that for the required return argument to be consistent with rational profit maximizing agents, the seller must have a reservation value that is sufficiently above that of the buying noise trader's.

A straight-forward extension would be to let noise traders be distributed over $\beta$. The required return would then depend on that distribution, and on the seller's need for funds. One could also envision a model where traders are two-dimensional in the sense that they have both an $\beta$ and a precision $\lambda$. This would open up for situations where informed traders may have to exit their positions prematurely. The next step would then be to endogenize also when this exit will take place. Such an endogenization would in turn open up for predatory traders. Such traders, discussed in Brunnermeier and Pedersen (2005), actively try to push other traders across their liquidity thresholds. Their objective is to be able to enter their positions at fire sale prices.

This type of two-dimensional individuals would also allow for the possibility noted by Gârleanu and Pedersen (2004), i.e. that a trader may trade for both informational or liquidity reasons, and that this affects the required return. We disregarded this possibility in the baseline model. However, we noted that their result depends on the information structure. The required return does decrease if the noise trader becomes informed by buying the asset. However, if he is informed already before buying the asset, then his opportunity cost changes, and the required return may in fact increase.

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